Causal Analysis of Probabilistic Counterexamples

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Motivation

Counterexample Analysis

❖ Inevitable complementary task to counterexample generation

❖ Error location is the most difficult part of debugging [Vesey]

Debugging Probabilistic Models

To answer the question:
Why is the probability threshold violated ?

Challenges for Analysing Probabilistic Counterexamples

❖ Multiple Paths

❖ Probabilistic Nature

Probabilistic Computation Tree Logic

PCTL Logic

PCTL is an extension of CTL for specifying probabilistic properties

State Formula $\phi ::= true|a|\neg\phi|\phi_1 \wedge \phi_2|\mathbf{P}_{\sim p}(\varphi)$

Path Formula $\varphi ::= \phi_1 \mathbf{U} \phi_2 | \phi_1 \mathbf{W} \phi_2 | \phi_1 \mathbf{U}^{\leq n} \phi_2 | \phi_1 \mathbf{W}^{\leq n} \phi_2$

PCTL Property Satisfaction

$$s \models \mathbf{P}_{\sim p}(\varphi) \Leftrightarrow P(s \models \varphi) \sim p$$

$$Pr(s \models \varphi) = Pr_s\{\pi \in Paths(s) \mid \pi \models \varphi\}$$

$$\sim \in \{<, \leq, >, \geq\}$$

Probabilistic Counterexamples

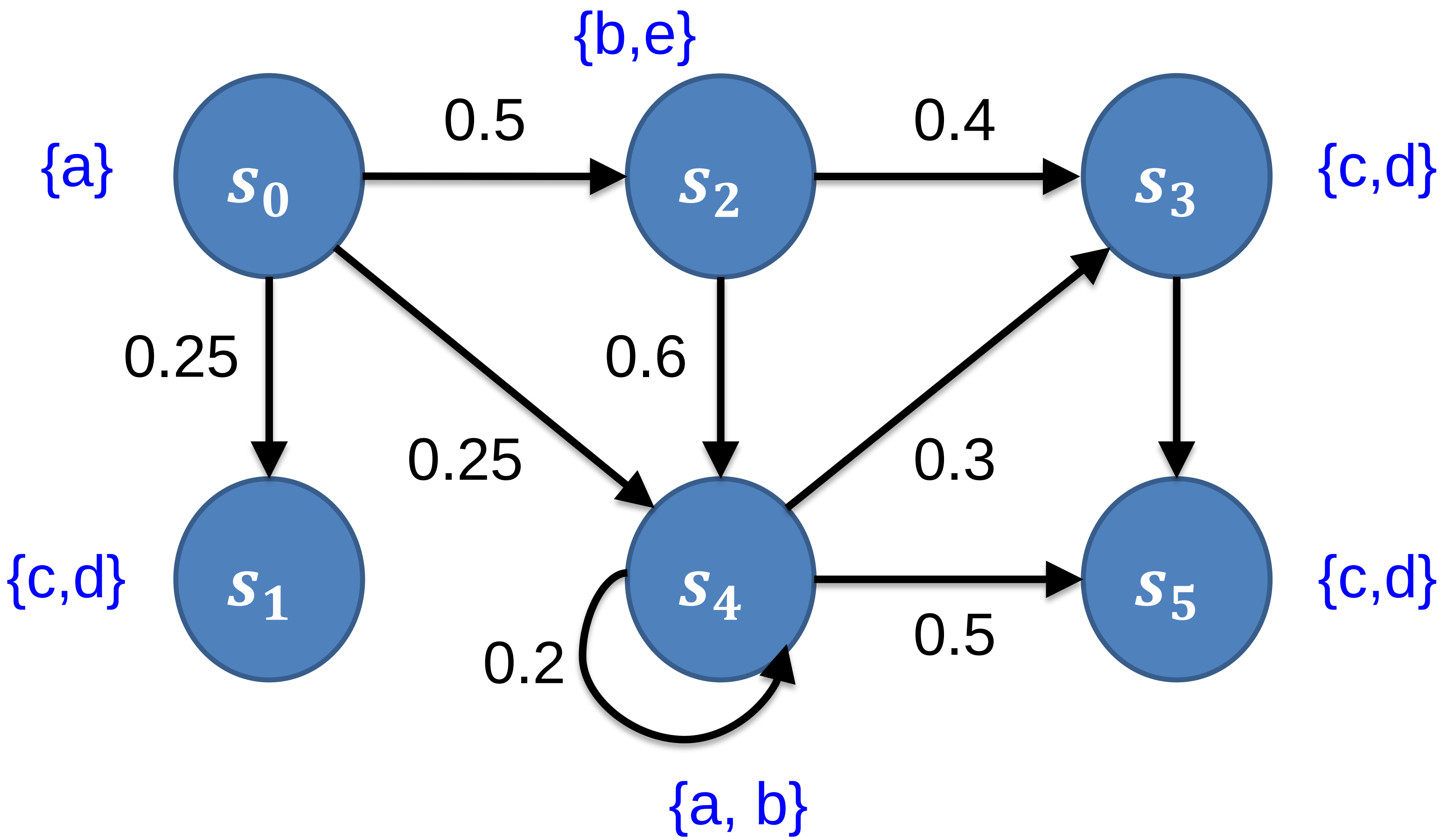
Probabilistic Counterexample

A counterexample C for $\boldsymbol{P}_{\leq p}(\boldsymbol{\varphi})$ is a set of finite paths with $\Pr(C) > p$

$s \nvDash P_{\leq 0.01}(F\ \text{error})$ ⟹ $\Pr(C) > 0.01$

Probabilistic Counterexamples

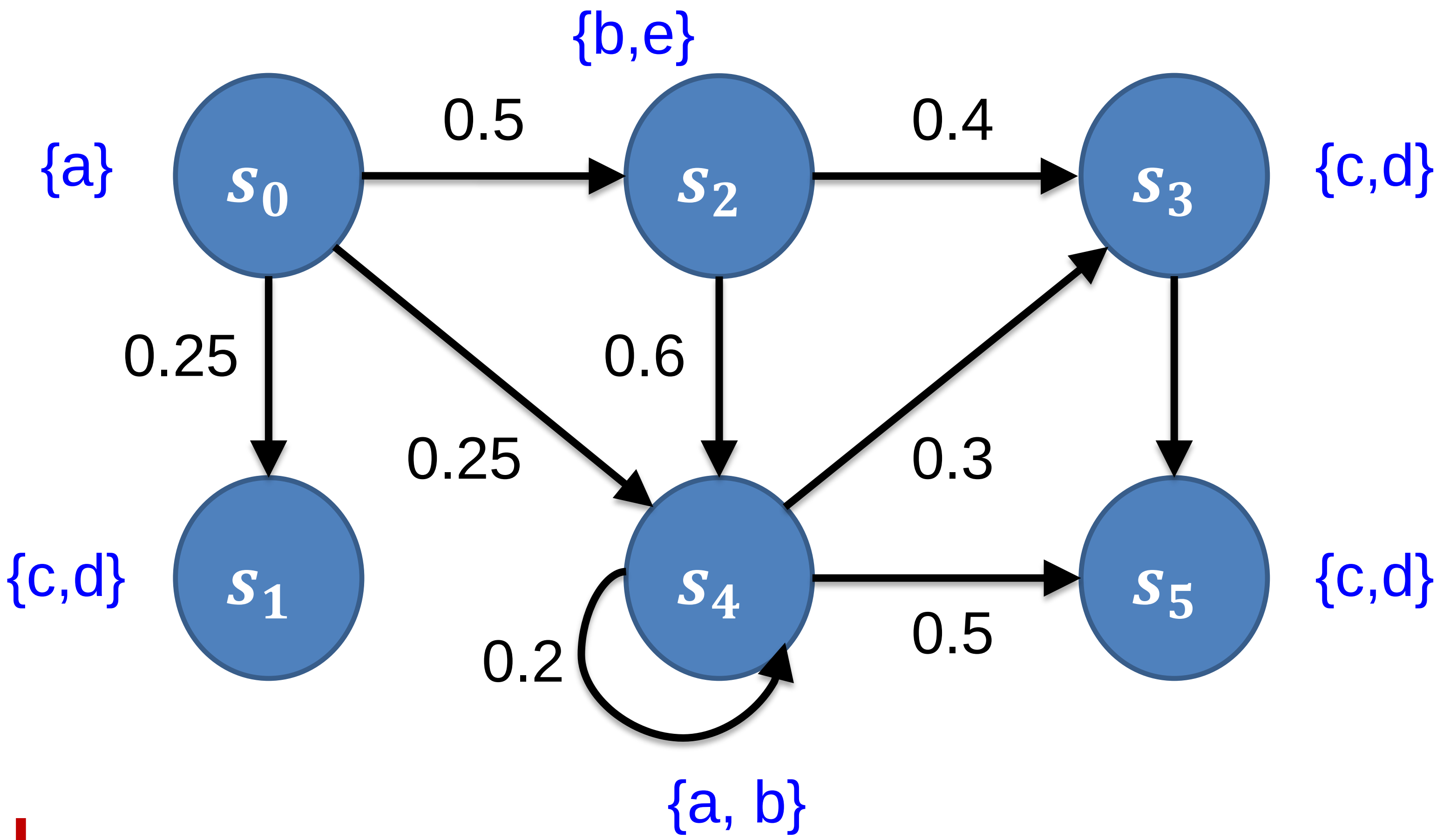
$$P_{\leq 0.5}[(a \vee b) U (c \wedge d)]$$



$$P(CX_2) = P(\{s_0s_1, s_0s_2s_3, s_0s_2s_4s_3, s_0s_2s_4s_5, s_0s_4s_5\})$$
$$= 0.25 + 0.2 + 0.09 + 0.15 + 0.12 = \mathbf{0.81}$$

Probabilistic Counterexamples

$$P_{\leq 0.5}[(a \vee b) U (c \wedge d)]$$

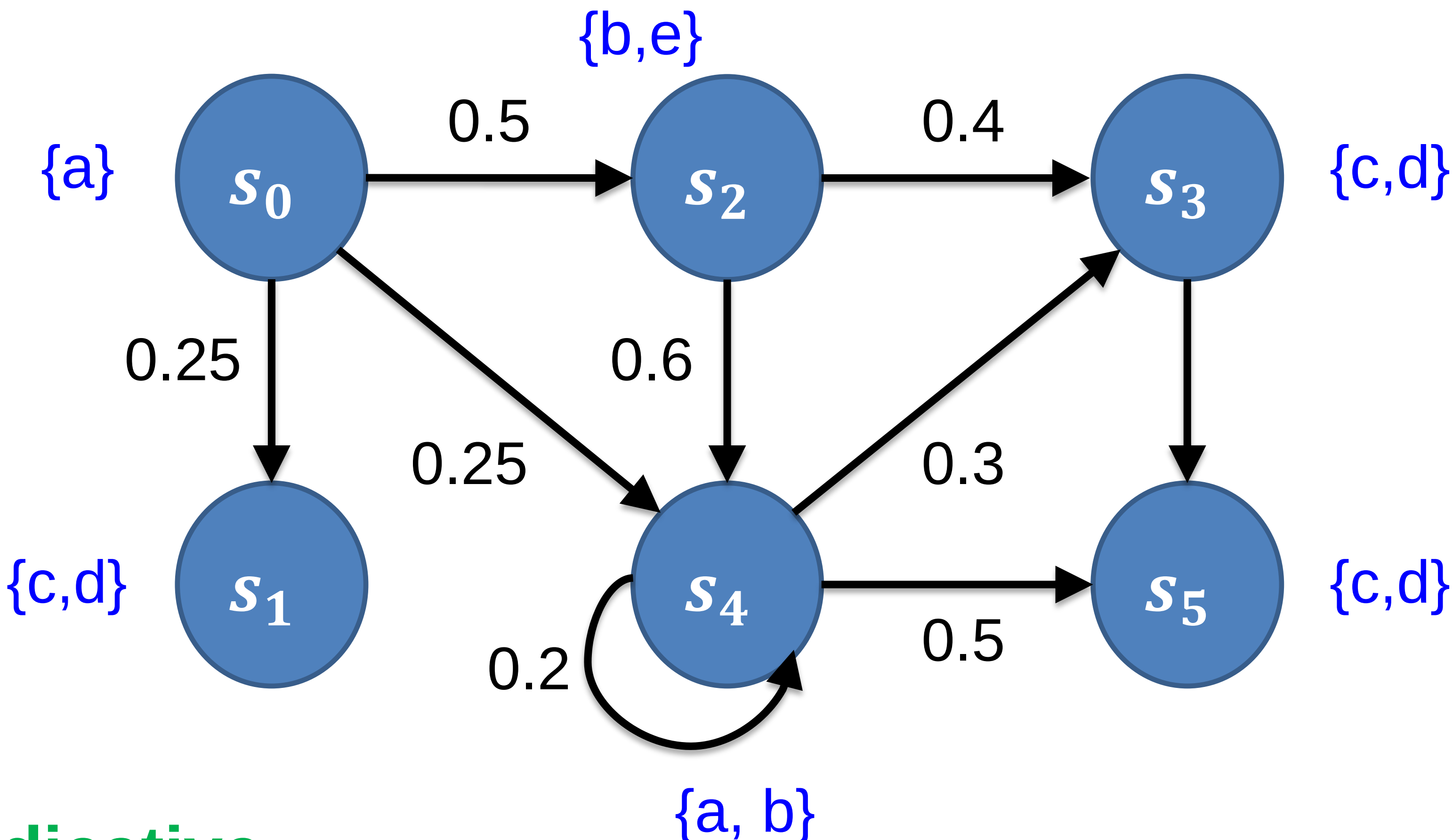


MINIMAL

$$P(CX_2) = P(\{s_0s_1, s_0s_2s_3, s_0s_2s_4s_3, s_0s_2s_4s_5, s_0s_4s_5\})$$
$$= 0.25 + 0.2 + 0.09 + 0.15 + 0.12 = \mathbf{0.52}$$

Probabilistic Counterexamples

$$P_{\leq 0.5}[(a \vee b) U (c \wedge d)]$$



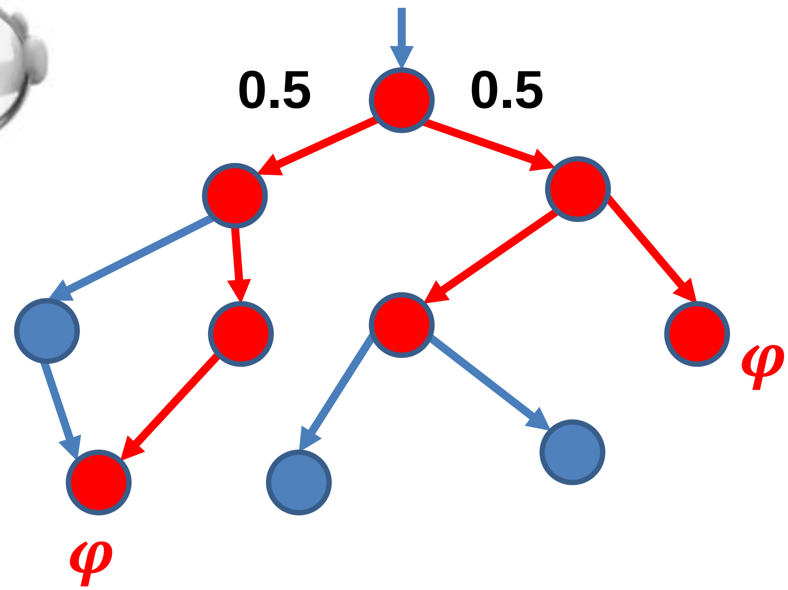
Most Indicative

$$P(CX_2) = P(\{s_0s_1, s_0s_2s_3, s_0s_2s_4s_3, s_0s_2s_4s_5, s_0s_4s_5\})$$
$$= 0.25 + 0.2 + 0.09 + 0.15 + 0.12 = \mathbf{0.60}$$

Counterexample Debugging

$MIPCX(s_0 \vDash \boldsymbol{\Phi})$

$\boldsymbol{\Phi} = P_{\leq p}(\varphi)$



Given

Most Indicative Probabilistic Counter Example (**MIPCX**)

Find

Labeling and probability values in the counterexample that cause the probability to exceed the given upper bound over the model

Causality and Responsibility for MIPCX

Criticality

$(s, X = x)$ is **critical**

if $\overbrace{MIPCX_{(s,X\leftarrow x')}}(s_0 \vDash \boldsymbol{\Phi})$ is not a valid counterexample.

$\overbrace{MIPCX_{(s,X\leftarrow x')}}(s_0 \vDash \boldsymbol{\Phi})$:

The set of finite paths resulting from $MIPCX(s_0 \vDash \boldsymbol{\Phi})$ by switching the value x of variable X in state s

Causality (adapted from Halpern & Pearl)

$(s, X = x)$is a **cause** for violating MIPCX

if either$(s, X = x)$is critical

or $W \leftarrow w'$ makes $(s, X = x)$ critical, for variable subset W

Degree of Responsibility (adapted from Chockler & Halpern)

$dR(s,\ X = x, \boldsymbol{\Phi}) = 1$ if $(s, X = x)$iscritical

$= 1/(|W| + 1)$ otherwise

Causality and Responsibility for MIPCX

Probabilistic Causality Model

is a tuple $< M,\ Pr >$
M : causality model and
Pr : probability function defined over the states of $MIPCX(s_0 \vDash \boldsymbol{\Phi})$

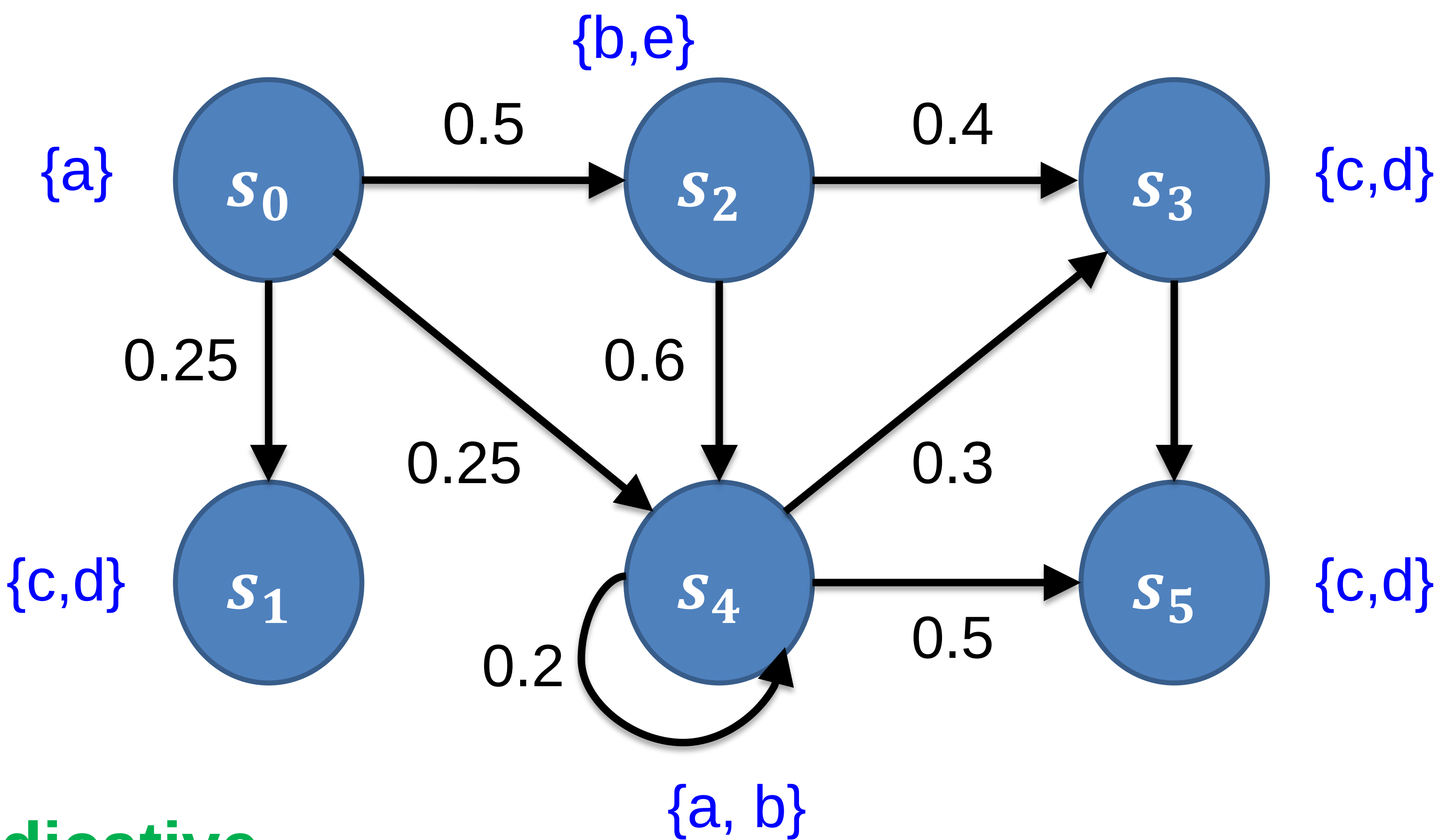
$$\Pr(s) = \sum_{s \in \sigma |\ \sigma \in MIPX(s_0 \vDash \boldsymbol{\Phi})} P(\sigma) \qquad \Pr(s, X = x) = \Pr(s)$$

Most Responsible Cause

Cause C is a most responsible cause for violating $\boldsymbol{\Phi} = P_{\leq p}(\varphi)$
if $dR(C)Pr(C) \geq dR(C')\Pr(C')$ for any cause C'.

Probabilistic Counterexamples Revisited

$$P_{\leq 0.5}[(a \vee b) U (c \wedge d)]$$

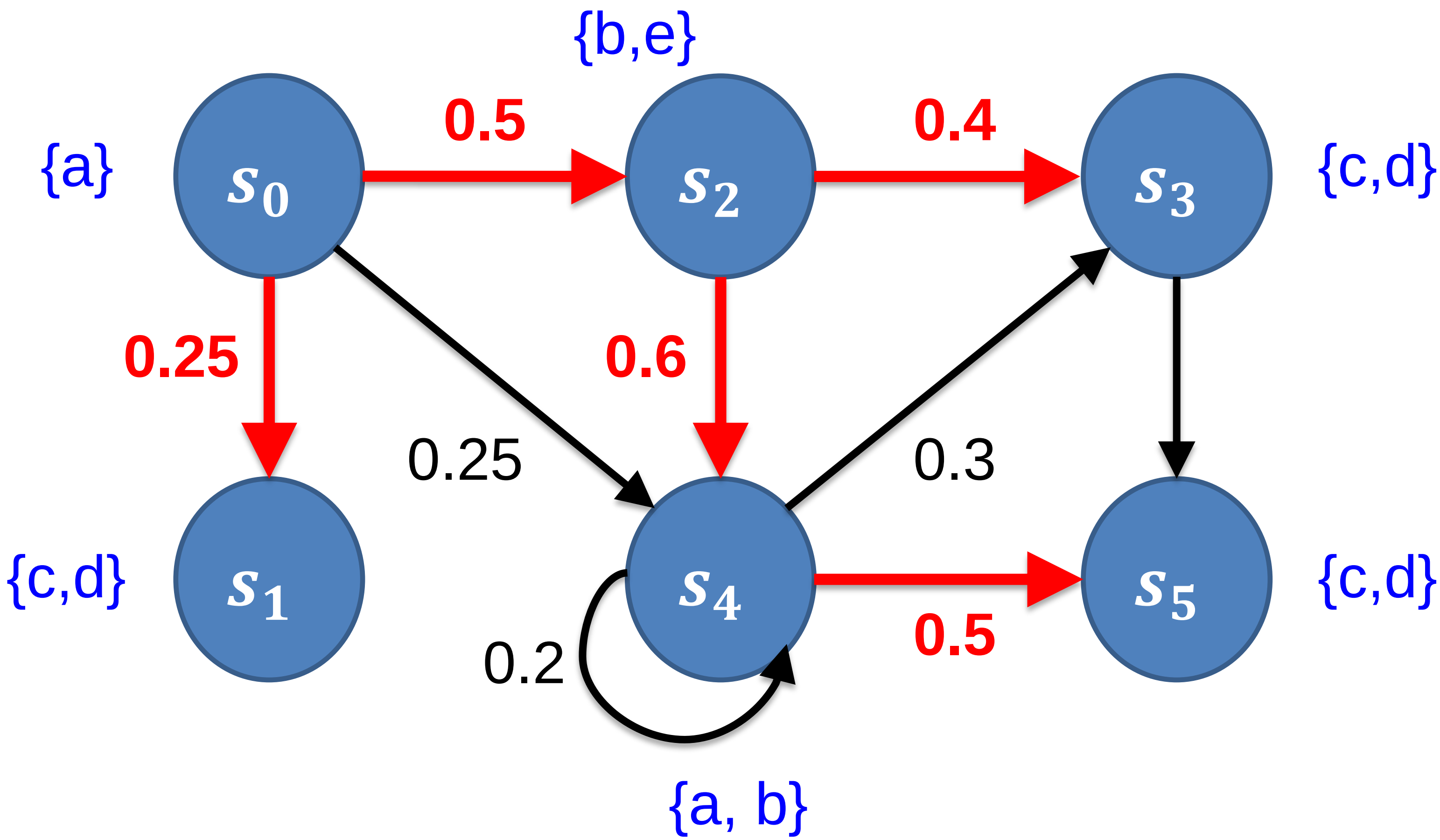


Most Indicative

$$P(CX_2) = P(\{s_0s_1, s_0s_2s_3, s_0s_2s_4s_3, s_0s_2s_4s_5, s_0s_4s_5\})$$
$$= 0.25 + 0.2 + 0.09 + 0.15 + 0.12 = \mathbf{0.60}$$

Probabilistic Counterexamples Revisited

$P_{\leq 0.5}[(a \vee b) U (c \wedge d)]$



$dR(s_4, b = 1) = 1/|\{a\}| + 1 = 0.5$

$dR(s_2, b = 1) = 1$

$Pr(s_2, b = 1) = 0.2 + 0.15 = 0.35$

$dR(s_2, b = 1)\Pr(s_2, b = 1) = 0.35$: highest

(s2,b=1) is the most responsible cause

Algorithm and Implementation

Probabilistic Symbolic Model Checker
[Kwiatkowska et al.]

Probabilistic Counterexample Generator
[Aljazzar et al.]

$MIPCX(s_0 \models \boldsymbol{\Phi})$
$\boldsymbol{\Phi} = P_{\leq p}(\varphi)$

Debugging Algorithm
(Debbi-Bourahla)

Diagnosis

Causes with
Responsibilities
and
Probabilities

Conclusion and Future Work

Conclusion

- We adapted and showed the usefulness of Causality and Responsibility in the context of debugging probabilistic counterexamples
- We introduced the notion of Most Responsible Cause as an indicator for the source of the error
- We developed a Debugging Algorithm, and tested it on real case studies with good performance

Future Work

- Visualization of diagnosis results